Pergamon

Int. J. Heat Mass Transfer. Vol. 38, No. 3, pp. 455–462, 1995

0017–9310/95 $9.50+0.00

0017–9310(94)00167–7

Analysis of one-dimensional fin assembly heat transfer with dehumidification

H. KAZEMINEJAD

Department of Mechanical Engineering, School of Engineering, University of Shiraz, Shiraz, Iran

(*Received in final form* 6 *June* 1994)

Abstract—Analysis of a one-dimensional conduction heat transfer model to study the performance of a cooling and dehumidifying fin assembly has been carried out. The conventional method was extended by incorporating the ratio of sensible to total heat transfer into the analysis of fin assemblies during dehumidification. The use of this ratio, which is widely used in psychrometric calculations, led to the identification of the correct qualitative importance of the parameters influencing the fin assembly performance during sensible heat transfer and sensible and latent heat transfer. The effects of variations in the relative humidity (50–70%), dry bulb temperature (25–35°C) and cold fluid temperature (0–10°C) on the temperature distribution and also on the augmentation factor, which are important during the design stage of finned tube dehumidifiers, are investigated and compared with those under dry conditions. It was concluded that a significant decrease in augmentation factor occurs as the amount of dehumidification increases.

INTRODUCTION

In conventional air-conditioning systems, finned tube heat exchangers are commonly used for air cooling and dehumidifying. Since, in most cooling coils, the coil surface temperature is below the dew point temperature of the air being cooled, simultaneous heat and mass transfer occur. Among the factors affecting the thermal performance of such heat exchangers are the geometry, materials and psychrometric conditions.

The design of dry finned surfaces is commonly based on a method of analysis in which the fins and the supporting interface are effectively treated as completely separate entities. First, the heat flow within the fins is analysed by examining the conductive heat flow within the fin and the convective heat dissipation from the surface of the fin. Then the effect of the thermal interaction between the supporting interface and the fins, and the convective heat exchange on the plain side of the supporting interface are incorporated by employing a technique based on electric circuit theory. The advantage of examining the fins in isolation from the supporting interface is that this permits a non-dimensional representation of the fin heat transfer rate, commonly referred to as the fin efficiency, which is widely regarded as the ideal design tool.

When simultaneous heat and mass transfer exist, a considerable change in the coil performance occurs, as a consequence of the change in the fin efficiency. Hence, the variation in the fin efficiency must be known to determine the overall cooling coil performance. When a phase change occurs, as in dehumidification, the corresponding change in fin efficiency becomes an influential factor in determining coil performance.

The overall performance of cooling coils with dehumidification has been widely studied in the past. Although the experimental data from various sources seem to be in general agreement, little theoretical work has been reported on condensation on fins or fin assemblies. Also, as far as the author is aware, it does not appear that any carefully controlled experiments on such geometries have been reported.

The importance of the understanding of the heat and mass transfer mechanisms on the fin performance of various geometries are reflected in numerous papers [1–5], reporting the results of specific investigations. However, techniques for the analysis of fin assembly heat transfer with only sensible heat transfer have been documented in a series of interesting papers published by Manzoor [6]. The most significant feature of his work is the introduction of an alternative to the fin efficiency, namely the enhancement factor. This enhancement factor is not as concise as the fin efficiency, in that it requires more charts in order to describe the fin side heat transfer. However, these charts clearly show the effects of variation in the fin dimensions, interfin spacing, fin thermal conductivity and surface heat transfer coefficient. Also, various aspects of the overall heat transfer rate are readily deduced from the enhancement factor. However, when simultaneous heat and mass transfer occur, no attempt has been made to analyse the fin heat transfer assembly.

In this study the theoretical method developed by Manzoor [6] has been extended for analysing the heat flow within finned surfaces when simultaneous heat and mass transfer occur, as in dehumidification processes. The use of the sensible to total heat transfer ratio, which is widely used in psychrometric calculations, and the appropriate dimensionless grouping

NOMENCLATURE

Aug	augmentation factor
Bi	Biot number, h_2t/k_f
Bi_1	cold fluid Biot number, h_1p/k_w
Bi_2	ambient Biot number, h_2p/k_w
h_1	cold fluid heat transfer coefficient
h_2	ambient heat transfer coefficient
h_{fg}	heat of condensation
h_m	mass transfer coefficient
k_f	fin thermal conductivity
k_w	wall thermal conductivity
K	k_f/k_w
l	fin length
L	aspect ratio, l/p
p	half-fin pitch
P	aspect ratio, t/p
q_l	latent heat flux
q_s	sensible heat flux
q_t	total heat flux, q_s+q_l
R	ratio of sensible to total heat transfer calculated at fin temperature
R_b	ratio of sensible to total heat transfer calculated at fin base temperature
R_l	ratio of sensible to total heat transfer calculated at fin tip temperature
RH	relative humidity
t	half-fin thickness
T	temperature distribution
T_1	cold fluid temperature
T_2	ambient dry bulb temperature
w	wall thickness
W	wall thickness aspect ratio, w/p
x	longitudinal displacement in wall
X	x/p
y	transverse displacement
Y	y/p
z	longitudinal displacement in fin
Z	z/p.

Greek symbols

ε	enhancement factor
θ	dimensionless temperature distribution, $(T-T_2)/(T_1-T_2)$
ρ_{ma}	moist air density
ω	moisture content.

Subscripts

1	plain wall
2	fin side
dry	without dehumidification
f	fin
w	wall
wet	with dehumidification.

Superscript

*	unfinned wall.

led to the identification of the correct quantitative importance of the parameters influencing the heat transfer performance of the fin assembly during sensible and latent heat transfer. The effects of variations in the dry bulb temperature, T_2, relative humidity, RH, and cold fluid temperature, T_1, on the temperature distribution which are not important under dry conditions have been presented graphically and compared with those under dry conditions. Also considered are the effects of variations in T_1, RH and T_2 on the ratio of dry to wet augmentation factor. The discussion that follows includes the governing equations and the results obtained for various parameters affecting the fin assembly heat transfer rate.

FORMULATION OF THE MODEL

When humid air contacts a surface at below its dew point temperature, the water vapour condenses on it in a filmwise, dropwise or mixed mode, depending on the condition of the surface. A clean surface tends to promote filmwise, and a treated surface dropwise, condensation. The presence of condensate on the cooling surface may enhance the heat and mass transfer at the surface due to increased roughness. The present analysis assumes that at the low RH encountered in practice, the condensate thermal resistance to heat flow is negligible because the condensate film is much thinner than the boundary layer in dehumidification processes. This assumption is based on earlier work investigated by the author [2, 3]. Under such circumstances, it is expected that the sensible heat transfer coefficient is not significantly influenced by the presence of condensation.

Consider a heat exchanger comprised of equally spaced longitudinal rectangular fins attached to a plane wall, as shown in Fig. 1. The following simplifying assumptions are made:

(1) The thermal conductivity of the fin and the wall are constant.
(2) The contact resistance between the wall and fin is negligible.
(3) The convective heat transfer can be described using Newton's law of cooling.
(4) The convective heat transfer coefficient over the fin side is constant.
(5) The conduction in the fin in the y-direction is negligible compared to that in the z-direction.
(6) The heat flow within the wall is one-dimensional even in the presence of the fins.
(7) Condensation occurs if the fin and the wall surface temperatures are below the dew point temperature of the surrounding humid air.

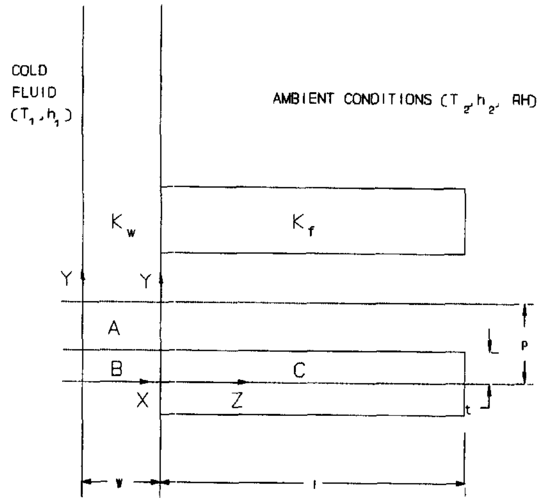

Fig. 1. Schematic representation of a fin assembly.

(8) The finned assembly is slightly tilted so that droplets can drain off the fin under the influence of the gravitational force.

These are essentially the classical assumptions inherent in the analysis of conducting–convecting finned surfaces.

Recent investigation [1] has indicated that assumption (4) is not strictly valid. However, an accurate representation of the convective heat transfer would require a simultaneous analysis of both the heat transfer and the fluid flow around the fins. The mathematical complexity of such a representation is beyond the scope of the present investigation.

The justification of assumption (5) is conventionally based on the criterion that the fin length be very much larger than the fin thickness. However, previous investigations [7] has shown that the applicability of the one-dimensional fin approximation is dependent upon the transverse Biot number, Bi, and not the ratio of the fin length to the fin thickness. The inference of both these criteria is that the sum-of-resistances method for predicting the overall heat transfer rate is applicable provided the heat flow within the fins is effectively one-dimensional. However, since the sum-of-resistances method neglects the two-dimensional effects induced within the wall by the presence of the fins [8], it is apparent that its applicability can only be justified by a direct comparison with the overall heat transfer rates predicted by a completely two-dimensional analysis. Therefore, no attempt is made in this study to qualify the applicability of the one-dimensional approximation. In particular, no approximations which require the fin length to be large in comparison with the fin thickness are introduced.

In order to develop a mathematical representation it is necessary to decide which section of the fin assembly should be examined. The geometrical symmetry of the fin assembly configuration, and the thermal symmetry resulting as a consequence of assumptions (2), (5) and (6), indicate that it will suffice to examine the heat flow within the region (A+B+C) (Fig. 1). This is the smallest region which includes all the essential features of the complete fin assembly, namely, the wall, fin and interfin spacing.

It may appear logical first to examine the heat flow within region A since, by virtue of assumption (6), the temperature distribution within this region will be identical to that within region B. However, if attention is restricted to region (B+C), then the effects of the interfin spacing will not be accounted for and therefore the model will not be representative of the fin assembly.

MATHEMATICAL ANALYSIS

On the basis of the foregoing assumptions, the determination of the temperature distribution within the fin assembly (and hence the heat transfer rate) requires the simultaneous solution of the following energy equations ([9]). Within the wall, regions A and B, the energy equation is

$$\frac{d^2\theta_w(X)}{dX^2} = 0 \tag{1}$$

and within the fin, region C, the fin temperature distribution can be obtained from the following equation:

$$\frac{d^2\theta_f(Z)}{dZ^2} - \frac{Bi}{P^2 R(\theta_f)}\theta_f(Z) = 0 \tag{2}$$

subject to the boundary conditions:

$$\text{at } X = 0, \quad \frac{d\theta_w(X)}{dX} = -Bi_1(1-\theta_w(X)) \tag{3a}$$

$$\text{at } X = W(Z=0), \quad \theta_w(X) = \theta_f(Z) \tag{3b}$$

and

$$\frac{d\theta_w(X)}{dX} = KP\frac{d\theta_f(Z)}{dZ} - \frac{Bi_2(1-P)\theta_w(X)}{R_b} \tag{3c}$$

$$\text{at } Z = L, \quad \frac{d\theta_f(Z)}{dZ} = -\frac{Bi\theta_f(Z)}{R_1 P} \tag{3d}$$

where R, the ratio of the sensible heat flux, q_s, to the total heat flux, $q_t = (q_s + q_l)$, is given by the following relation:

$$R(\theta) = \frac{q_s}{q_t} = \left[1 + (\rho_{ma}h_{fg})(T_2 - T_1) \times \left(\frac{h_m}{h_2}\right)\left(\frac{\omega_2 - \omega(\theta)}{\theta}\right)\right]^{-1}. \tag{4}$$

This ratio is widely used in psychrometric calculations and its value can lie between zero and unity when simultaneous heat and mass transfer occur. For details of the calculation of R see ref. [10].

Boundary conditions (3a) and (3d) describe the convective heat exchange at the plain side of the wall and the tip of the fin, respectively. Boundary conditions (3b) and (3c) arise as a consequence of the perfect contact and continuity of temperature and heat flux across the wall-to-fin interface.

The solution to the problem described by equations (1) and (2) and boundary conditions (3) when only sensible heat transfer occurs (i.e. $R = 1$) is [6]

$$\theta_w(X) = \frac{-X + W + \dfrac{\varepsilon}{Bi_2}}{\dfrac{1}{Bi_1} + W + \dfrac{\varepsilon}{Bi_2}} \tag{5}$$

and

$$\theta_f(Z) = \frac{\dfrac{\varepsilon}{Bi_2}}{\dfrac{1}{Bi_1} + W + \dfrac{\varepsilon}{Bi_2}} \times \left[\frac{\cosh(1 - Z/L)\dfrac{L}{P}Bi^{1/2} + (Bi)^{1/2}\sinh(1 - Z/L)\dfrac{L}{P}(Bi)^{1/2}}{\cosh\dfrac{L}{P}(Bi)^{1/2} + (Bi)^{1/2}\sinh\dfrac{L}{P}(Bi)^{1/2}}\right] \tag{6}$$

where

$$\varepsilon = \frac{Bi_2}{Bi_2(1 - P) + K(Bi)^{1/2} \times \left[\dfrac{\sinh\dfrac{L}{P}(Bi)^{1/2} + (Bi)^{1/2}\cosh\dfrac{L}{P}(Bi)^{1/2}}{\cosh\dfrac{L}{P}(Bi)^{1/2} + (Bi)^{1/2}\sinh\dfrac{L}{P}(Bi)^{1/2}}\right]}. \tag{7}$$

The solution to the problem described by equations (1) and (2) and boundary conditions (3) precludes an analytical treatment when dehumidification occurs because R is a function of θ and the full set of equations as posed are inhomogeneous. Therefore, in order to determine the temperature distribution a numerical technique is devised. The equations are first reduced to a system of first-order differential equations and are solved numerically using a shooting method which combines the classical Runge–Kutta method and the Newton–Raphson iteration. The accuracy of the numerical solution procedure was established by comparing the dry condition results (i.e. for $R = 1$) with those obtained from equations (5) and (6).

FIN ASSEMBLY HEAT TRANSFER RATE

The heat flow through the fin assembly is conveniently expressed in the form of an augmentation factor, *Aug*, defined as the ratio of the heat transfer rate of the finned assembly to that of the unfinned wall operating under the same conditions. In order to evaluate *Aug* it is first necessary to determine the heat flow rate through the unfinned wall. This requires the solution of the energy equation within the wall:

$$\frac{d^2\theta_w(X)}{dX^2} = 0 \tag{8}$$

subject to the boundary conditions:

$$\text{at } X = 0, \quad \frac{d\theta_w(X)}{dX} = -Bi_1(1 - \theta_w(X)) \tag{9a}$$

$$\text{at } X = W, \quad \frac{d\theta_w(X)}{dX} = -\frac{Bi_2\theta_w(X)}{R_b} \tag{9b}$$

where the differential equation (8) is obtained by performing an energy balance on an infinitesimal element of the wall, and boundary conditions (9a) and (9b) described the convective heat exchange from the surface of the wall. The solution to this problem when only sensible heat transfer occurs ($R_b = 1$) is

$$\theta_w^*(X) = \frac{-X + W + \dfrac{1}{Bi_2}}{\dfrac{1}{Bi_1} + W + \dfrac{1}{Bi_2}} \tag{10}$$

and, therefore,

$$Aug = \frac{\dfrac{1}{Bi_1} + W + \dfrac{1}{Bi_2}}{\dfrac{1}{Bi_1} + W + \dfrac{\varepsilon}{Bi_2}}. \tag{11}$$

RESULTS AND DISCUSSIONS

From equation (7) it can be deduced that the enhancement factor ε may be parameterised by the Biot number, Bi_2, the ratio of the thermal conductivity, K, and the aspect ratios L and P (since $Bi = Bi_2P/K$). The behaviour of the enhancement factor with variations in these parameters has been extensively examined by Manzoor [6] with a view to understanding the effects of changes in the fin parameters on the overall heat transfer rate when only sensible heat transfer occurs. However, when simultaneous heat and mass transfer occur, as in dehumidification processes, it is not possible to develop an analytical expression similar to equation (7) to determine the enhancement factor due to the complexity of the analysis and the inhomogeneous nature of the full set of differential equations.

In this study, the effects of variations in T_2, RH and T_1 are examined to provide a greater understanding of the simultaneous heat and mass transfer which occur in industrial dehumidifiers.

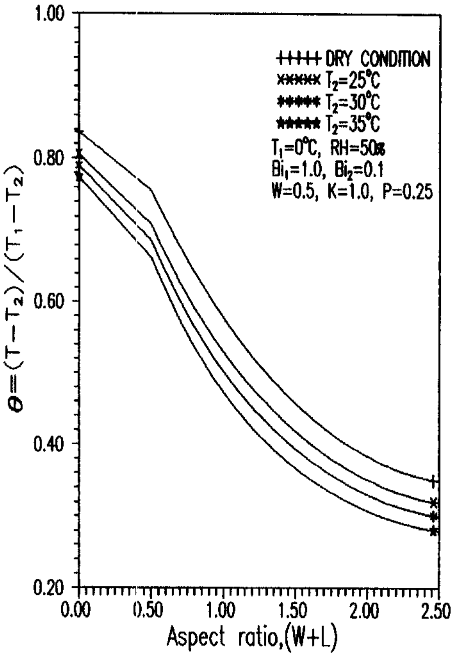

Fig. 2. Effect of dry bulb temperature, T_2, on the temperature distribution.

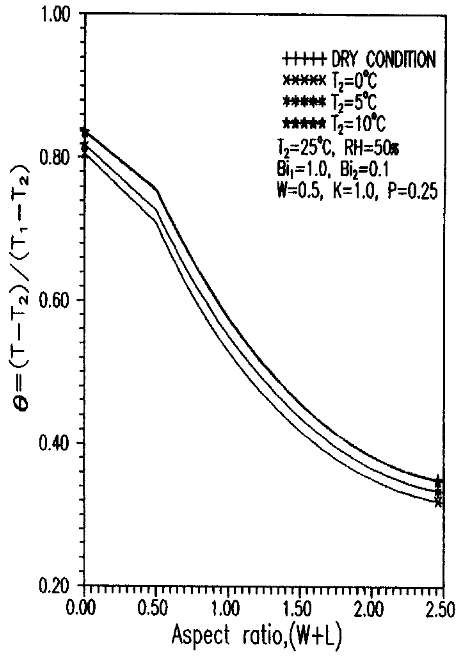

Fig. 4. Effect of cold fluid temperature, T_1, on the temperature distribution.

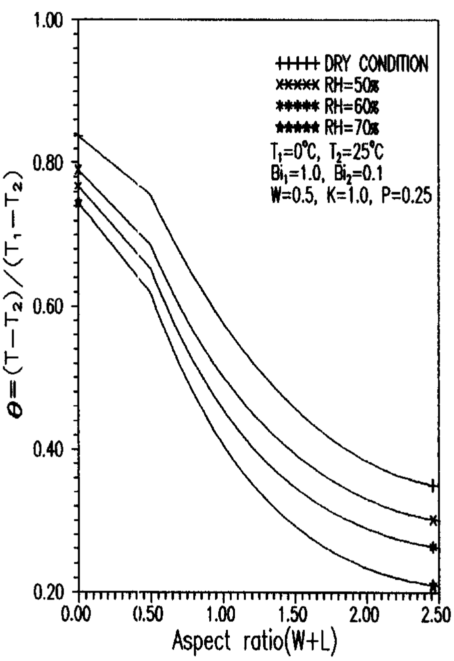

Fig. 3. Effect of relative humidity, RH, on the temperature distribution.

Results are given in Figs. 2–7. In the graphs presented in Figs. 2–7, the parameters Bi_1, Bi_2, K, P and W are assigned the following prescribed values:

Bi_1	Bi_2	K	P	W
1.0	0.1	1.0	0.25	0.5

These values represent the performance of a finned heat exchanger with forced convection of cold liquid on the plain side and free convection of humid air on the fin side. In Figs. 2–4, the dimensionless temperature distribution, θ, is plotted against the aspect ratios $(W+L)$ for three different values of T_2, RH and T_1. Also considered is the effects of variations in T_1, RH and T_2 on the ratio of the dry to the wet augmentation factor, $(Aug)_{\text{dry}}/(Aug)_{\text{wet}}$ (Figs. 5–7). The features of these results are discussed below.

Effect of T_2

Shown in Fig. 2 is θ plotted against the aspect ratios $(W+L)$ for both dry and wet conditions, and for various dry bulb temperatures. It can be seen that the curves for a fin assembly with dehumidification lie below that of a dry fin assembly. As the dry bulb temperature increases, the departure of θ from the dry surface values becomes greater. This is attributed to the increase of the temperature and to moisture content differences between the bulk and the fin surface, which consequently leads to higher sensible and latent heat transfer, other conditions being kept constant.

Effect of RH

One of the special features of the present results illustrated in Fig. 3 is the variation of RH from 50 to 70%. It shows that as RH is increased, θ is decreased and the departure of the temperature profiles from the dry surface curve becomes greater. The increase of the air RH is accompanied by an increase in the motive force of water vapour diffusion, and hence by the number of water vapour molecules condensing on the

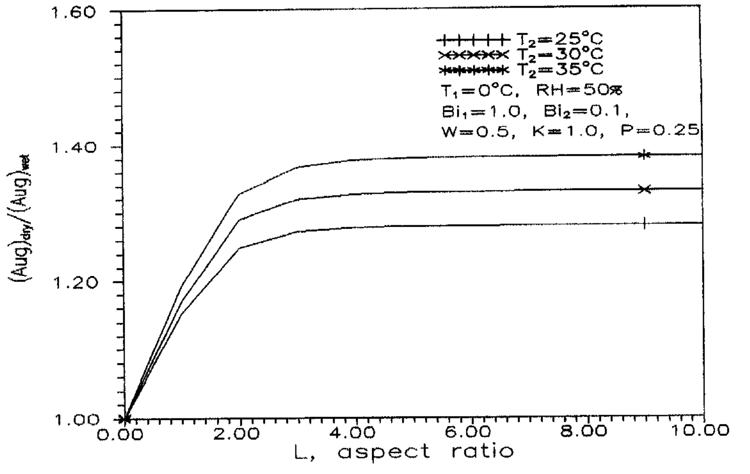

Fig. 5. Effect of dry bulb temperature, T_2, on $(Aug)_{dry}/(Aug)_{wet}$.

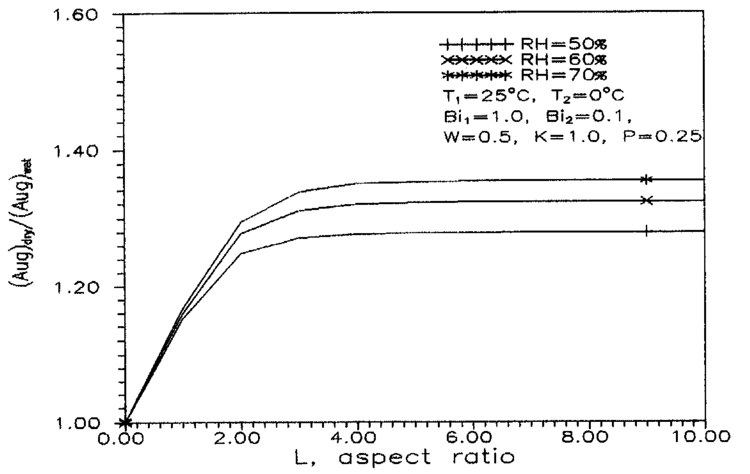

Fig. 6. Effect of relative humidity, RH, on $(Aug)_{dry}/(Aug)_{wet}$.

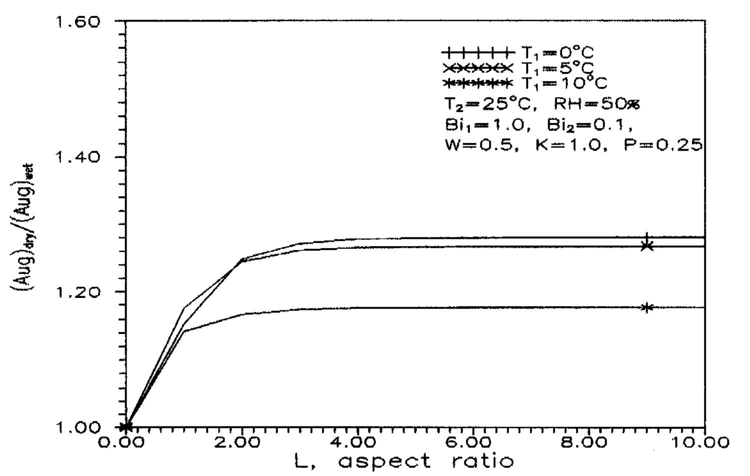

Fig. 7. Effect of cold fluid temperature, T_1, on $(Aug)_{dry}/(Aug)_{wet}$.

fin surface. This results in a higher latent heat transfer and a higher fin surface temperature.

Effect of T_1

The increase in T_1 (Fig. 4) increases the fin surface temperature and hence decreases the driving potential for both heat and mass transfer. Although the values of θ for $T_1 = 0$ and 5°C are lower than those under dry condition, the difference between the values at $T_1 = 0$ and 5°C is not significant. In addition, It is interesting to note that, on increasing the value of T_1, to 10°C, no condensation occurs and the wet and the dry temperature distributions coincide. Thus, as would be expected, increasing T_1 above the dew point temperature of the surrounding humid air facilitates an increase in the overall heat flow rate from the fin assembly.

Comparison of dry and wet fin assembly heat transfer, $(\mathrm{Aug})_{dry}/(\mathrm{Aug})_{wet}$

Computation of the heat transfer performance of a fin assembly with and without dehumidification were performed for three values of T_1, RH and T_2, other conditions being constant. These results are shown in Figs. 5–7, respectively. The effects of variations in the psychrometric conditions on the overall heat transfer rate can be deduced by plotting the ratio of $(Aug)_{\mathrm{dry}}/(Aug)_{\mathrm{wet}}$ against L. Figures 5–7 show that the augmentation ratio increases rapidly with an increase in L. It then reaches a limiting value as L is further increased. In addition, Figs. 5–7 show that the augmentation ratio increases appreciably during dehumidification and is further increased with an increase in T_2 and RH, or a decrease in T_1. At smaller values of L, the sharp increase in the augmentation ratio is due to the increase in the fin assembly surface temperature.

One of the special features of the results shown in Figs. 5–7 is the manner in which the augmentation factor approaches some limiting value as the fin aspect ratio, L, is increased. This limiting value for the case when $R = 1$ can be determined analytically by examining the behaviour of enhancement factor, ε, for large values of L. Expression (7) for ε can be expressed as

$$\varepsilon = \frac{1}{(1-P)+\left(\frac{KP}{Bi_2}\right)^{1/2}\left(\frac{\tanh\frac{L}{P}Bi^{1/2}+Bi^{1/2}}{1+Bi^{1/2}\tanh\frac{L}{P}Bi^{1/2}}\right)}. \quad (12)$$

Therefore, for large values of L,

$$\varepsilon \sim \frac{1}{(1-P)+\left(\frac{KP}{Bi_2}\right)^{1/2}}. \quad (13)$$

This tendency of Aug to approach some limiting value indicates that the overall heat transfer cannot be indefinitely increased by increasing the fin length. It is not immediately apparent why this should be the case. However, an explanation can be formulated from a physical viewpoint. The addition of fins to a plane wall results in an increase in the total heat transfer surface area, but at the same time introduces an additional conductive resistance. Initially the gain in surface area far outweighs the extra conductive resistance. However, eventually a state is reached such that further increase in the fin length is negated by the respective increase in the conductive resistance. In all cases considered, the results presented in Figs. 5–7 agree with this limiting value of the augmentation factor.

CONCLUSIONS

The performance of a cooling and dehumidifying fin assembly heat transfer has been studied when simultaneous heat and mass transfer occur. The effects of variations in T_1, RH and T_2 on its heat transfer characteristics have been investigated. From the previously discussed results, the following conclusions emerge:

(1) The heat transfer rate to a fin assembly when dehumidification occurs is remarkably higher than that of a dry fin assembly and this is due to the additional latent heat transfer during the condensation process.
(2) A significant reduction in the wet augmentation factor occurs when the amount of dehumdification increases. The reduction of the wet augmentation factor from that with a dry surface condition depends on the differences in T_2 and specific humidity between the surrounding humid air and the fin assembly interface.
(3) The augmentation factor of a wet fin assembly is significantly influenced by T_1 compared to the dry surface condition. As T_1 is increased to the dew point temperature of the surrounding humid air, the ratio of the dry to the wet augmentation factor approaches the value of unity.
(4) The ratio of the dry to the wet augmentation factor increases sharply with an increase in L. It also increases with an increase in T_2 and RH. However, it decreases with an increase in T_1, other conditions being kept constant. At an aspect ratio of approximately 4.0, it approaches some limiting value as the fin length is increased.

The mathematical model presented is sufficient to predict the heat transfer performance of a fin assembly when dehumidification occurs. This formulation in itself represent a major extension of the previous work, as the previous studies in this area have restricted attention solely to the fin side and not the complete fin assembly, i.e. both the fins and the supporting surface simultaneously. The results can be used together with the previous results provided in ref. [6] during the design stage to determine the optimum fin assembly for dehumidification purposes. The general fin assembly situation, on the basis of two-dimensional heat flow, is currently under investigation.

Acknowledgement—This research is supported by the University of Shiraz (grant No. 69-EN-584-311) whose assistance is hereby gratefully acknowledged.

REFERENCES

1. H. Kazeminejad, M. A. Yaghobi and F. Bahri, Conjugate forced convection–conduction analysis of the performance of a cooling and dehumidifying vertical rectangular fin, *Int. J. Heat Mass Transfer* **36**, 3625–3631 (1993).
2. J. E. R. Coney, H. Kazeminejad and C. G. W. Sheppard, Dehumidification of air on vertical rectangular fin: a numerical study, *Proc. Instn Mech. Engrs Part C* **203**, 165–175 (1989).
3. J. E. R. Coney, H. Kazeminejad and C. G. W. Sheppard, Dehumidification of turbulent air flow over a thick fin: an experimental study, *Proc. Instn Mech. Engrs Part C* **203**, 177—188 (1989).
4. A. Kilic and K. Onat, The optimum shape for convecting rectangular fin when condensation occurs, *Wärme- und Stoffubertragung* **15**, 125–133 (1981).
5. M. Torner, A. Kilic and K. Onat, Comparison of rectangular and triangular fin when condensation occurs, *Wärme- und Stoffubertragung* **17**, 65–72 (1983).
6. M. Manzoor, *Heat Flow through Extended Surface Heat Exchangers*, Lecture Notes in Engineering. Springer-Verlag (1984).
7. W. Lau and C. W. Tan, Errors in one dimensional heat transfer analysis in straight and annular fins, *J. Heat Transfer* **95**, 549–551 (1973).
8. N. V. Suryanarayana, Two dimensional effects on heat transfer from an array of straight fins, *J. Heat Transfer* **99**, 129–132 (1977).
9. F. P. Incropera and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer* (2nd Edn), pp. 62–103. Wiley, New York (1985).
10. H. Kazeminejad, Dehumidification of turbulent air flow over a thick fin, Ph.D. thesis, University of Leeds (1987).